

INTERNAL FRICTION IN METALS

A DISSERTATION

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BY

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ABSTRACT

A tubular specimen of the metal is made the elastic control of a heavy pendulum bar. The system can be set into forced vibrations, either bending or twisting the specimen, by a measured periodic magnetic force-couple. The resonance amplitude and frequency, with the moment of the impressed force-couple, yield the ratio of the energy dissipated in one cycle to the energy of the vibration. The tubular form of the specimen enables the relation between dissipation and stress-amplitude to be studied in detail, at least for torsion. For normal (unfatigued) metal, the internal friction appears to be entirely associated with shear. This is not borne out in certain specimens of fatigued or overstrained metal; but these exceptions are easily accounted for. If the energy dissipated per cc. per cycle of stress-amplitude is represented by F , the relation of friction to stress-amplitude is given by the formula $F = 8\phi W (1 - f_0/f)$ where W is the total strain energy, ϕ is the coefficient of internal friction, and f_0 is a "threshold" stress-level which is zero initially, but takes on larger values as the vibration history becomes longer. If $f_0 = 0$, this formula is the same as that proposed by Kimball, viz. $F = \xi f^2$. The coefficient ϕ here introduced is independent of systems of units and to a certain extent independent of the vibration history. If f is less than f_0 the dissipation is negligible; if f is greater than another much higher level f_1 , F increases faster than any power of f . Repeated cycles of high amplitudes cause all the constants gradually to change; f_0 decreases again, ϕ increases many fold, the upper limit itself may change. These progressive changes are probably associated with the progress of fatigue.

INTRODUCTION

PREVIOUS workers, using various methods, have found values of the internal friction (often considered to be viscosity) which are not in marked agreement, even as to order of magnitude. Kimball and Lovell¹ have done much to clarify the situation by showing: (1) that the dissipative forces are independent of the strain-velocity, hence not to be described as due to viscosity; (2) that at least for certain stress-ranges the work dissipated is proportional to the square of the stress-amplitude.

The method used by previous workers has most often been to determine the rate of decay of elastic vibrations of one type or another, though Kimball's very important results were obtained by an accurate measurement of the energy lost in rotating a bent rod. In nearly every case the experiments have been open to two objections: (1) the small size of the specimens, which exaggerates errors dependent on surface conditions, and (2) the use of solid specimens in which different portions of the metal pass through very different stress-cycles. Results will be presented in this paper which show that the behavior of the metal depends on the stress-amplitude even for very small

¹ Kimball and Lovell, Phys. Rev. 30, 6 (1927).

values of the latter, and that the process of fatigue produces changes which can only be studied in a homogeneously strained specimen.

APPARATUS

Fig. 1 is a schematic view of the apparatus used. The specimen *S* (shown enlarged in the upper right hand corner) is of hollow section, about one cm inside diameter and 1.2 cm outside. At the ends it is much enlarged and threaded to screw into the pendulum bar above and the heavy base-bar below. Both these bars have longitudinal slots, and are provided with bolts by which the specimen is rigidly clamped in place. No evidence of slipping could be detected. The base-bar is in turn bolted to a concrete pier weighing about 200 kilos.

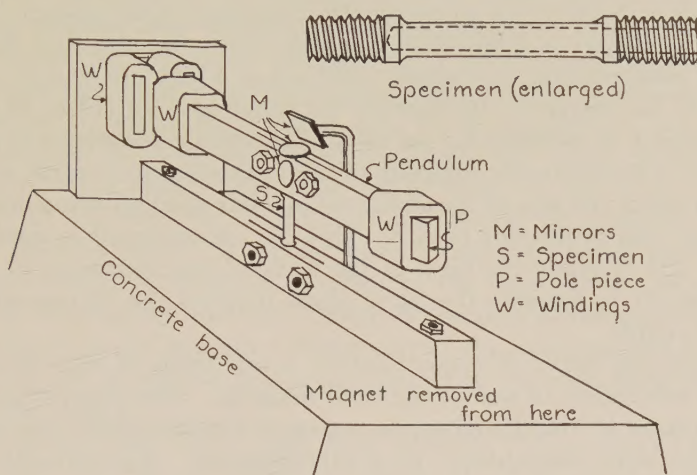


Fig. 1. Diagram of apparatus

The pendulum bar, 40 cm long, and weighing about 8 kilos, is made of cast iron. It carries coils *W* at its ends, as well as wedge-shaped pole-pieces *P*, which may be rotated so that their edges are horizontal. At the ends of the base-bar are standards supporting U-shaped electromagnets. These are so arranged that a pole piece of the pendulum is between the two poles of each magnet. One of these has been omitted in the figure, to permit a better view of the pendulum. These latter magnets are likewise capable of rotation so that their edges are horizontal.

It is plain that if both these magnets are maintained at a constant strength, while an alternating current is passed through the pendulum windings, a periodic impulse-couple will be exerted on the pendulum. This couple will produce torsion of the specimen if the arrangements are as shown; but if the fixed magnets and pole pieces are turned to a horizontal position the couple will produce bending.

Vibrations set up in the pendulum are measured by means of concave mirrors, *M*, which reflect the image of a straight lamp filament on a scale. For torsion the mirror is placed on the side of the pendulum; for bending

it is placed horizontally on top and used in conjunction with a plane mirror tilted at an angle of 45° . The scale distance is 122 cm, and the amplitude of the band of light ranges up to 10 cm, and may be measured to 0.1 mm.

The remainder of the apparatus consists of a motor driven commutator for producing the required alternating current, together with resistances and meters for controlling and measuring the frequency and intensity of the alternating current and the intensity of the direct current in the fixed magnets.

The motor (1/6 hp) is connected through springs to a long shaft which carries three commutators. One of these, very accurately made, with carbon brushes and protective condensers, is used to reverse the pendulum current. A 120 volt d.c. laboratory circuit supplies all the power used. The second commutator is the means of measuring the frequency of the system. Through it a large condenser is alternately charged from a battery and discharged through a milliammeter, the steady readings of which are found to be proportional to the frequency throughout the range covered (15 to 30 per sec.) with a precision of perhaps 0.2 per cent. The third commutator is provided with a pair of diametrically opposite brushes attached to a frame which can be rotated about the axis of the shaft, the angular position being determined by an index and protractor to 0.5° . This last arrangement is used for two purposes: (1) to study the form of the wave of magnetization in the pendulum, and (2) to measure the lag in phase between the vibrations and the impressed couple.

To study the curve of magnetization, a few turns of wire were passed round the pole pieces of one of the fixed magnets. The alternating current induced in these by the changing magnetism of the pendulum was conducted through the third commutator to a galvanometer. The movable pair of brushes was placed at various angles, and the corresponding galvanometer

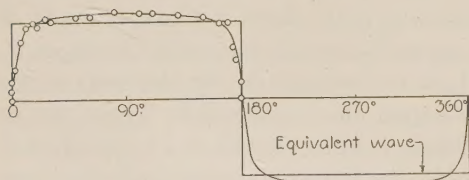


Fig. 2. Form of wave of magnetization.

readings recorded. These readings can be shown to be proportional to the total magnetic flux through the coil at that part of the wave of magnetization corresponding to the brush setting. In this way the curve shown in Fig. 2 was obtained. It is seen to be practically a square topped wave. In fact, since the experiments were made at resonance frequency, where the lag between vibration and impulse-wave is 90° , most of the work is done by the couple when it has its most nearly constant value.

Finally it may be worth mentioning that the speed of the motor is controlled by supplying its armature, separately from the field, with current drawn from a potentiometer carrying two amperes. From this it passes through a carbon compression rheostat with a spring under the screw. This device permits the extremely precise speed control demanded in the experiment.

CONSTANTS AND CORRECTIONS

The moment of inertia of the pendulum for torsion is of fundamental importance. It was determined by two independent methods, involving nothing novel, and the result is considered correct to 0.2 percent.

In the case of the bending vibrations the center of oscillation is at a point about half way down the specimen. This point was exactly located by calculations from elastic theory. From the location of this point it was then easy to deduce the moment of inertia for bending vibrations.

The most difficult to determine, and perhaps the most doubtful, constants of the apparatus are those which determine the strength of the couple exerted when known currents pass through the windings. The method used was to measure photographically the amplitude of the static deflections produced by known couples, also measured photographically. These couples were then multiplied by a constant less than unity determined from the form of the magnetization curve described above. The results were a series of curves giving factors by which the product $I_f I_m$ of the currents in the fixed and moving magnets is to be multiplied to obtain the torque exerted. This factor varied from about 1.3×10^6 to 2×10^6 dyne-cm per ampere squared.

Finally to ascertain the dissipative forces within the specimen, we must eliminate those outside it. It was established that there was negligible vibration, hence negligible dissipation, either in the base-bar or the pendulum itself, or at the clamped surfaces. About 80 percent of the whole amplitude is due to strain in the straight tubular part of the specimen and for various reasons it is safe to assume that a like portion of the total dissipation takes place in this section.

There remain two ways other than internal friction by which energy may be absorbed: the air, either by friction or by radiation of sound; and eddy currents set up in the solid cores of the magnets. The correction for air was established by attaching to the pendulum a light duplicate pendulum some distance above the original, and observing how this increased the total dissipation of energy. The correction for eddy currents was arrived at by comparing the rates of decay of free vibrations when the magnets were excited and unexcited. The result was two correction-formulas which affect the final results only by a small percentage.

These corrections are perhaps negligible compared to other uncertainties in the experiment, but were in all cases duly allowed for.

THEORY

The theory of forced vibrations of the type to be considered here is better developed from energy equations than from a differential equation. We picture the pendulum vibrating at a steady amplitude while acted upon by a periodic force-couple of magnitude $\pm L$, the $\pm$ sign indicating that the couple is suddenly reversed twice during each period. This assumption corresponds to the square-topped wave of magnetization found to exist in the apparatus as used. Let the frequency be n and let the pendulum make harmonic vibrations according to the equation: $\theta = a \sin(2\pi nt - \alpha)$, where

θ = angular displacement, a is the amplitude, t the time, and α the lag in phase between impressed couple and vibration. Then the work done by the external couple in one complete vibration is given by

$$\text{Work} = 4La \sin \alpha \quad (1)$$

Evidently the work done on the specimen is a maximum when $\alpha = \pi/2$.

Let the energy dissipated by internal friction during each vibration be F , where F is a function of the amplitude only and increases with the amplitude. We then have

$$4La \sin \alpha = F(a) \quad (2)$$

If F is a function of degree higher than unity, a will have its maximum value for a given L when $\alpha = \pi/2$, i.e., when the external work is a maximum. Finally, where A is the amplitude at resonance,

$$4LA = F(A) \quad (3)$$

Let us now revert to the specimen itself. Assuming for simplicity's sake the existence of only one type of stress, e.g., shear, the elastic strain-energy w per unit volume is given by $w = f^2/2E$, where f is the stress amplitude and E the appropriate elastic modulus. The total elastic energy W of the specimen will be

$$W = \iiint f^2 dv / 2E \quad (4)$$

where the integration extends throughout the volume of the specimen. This is the same as the potential energy of the pendulum at maximum displacement, which is the same as its kinetic energy at mid-swing. Hence

$$W = 2\pi^2 I n^2 a^2 \quad (5)$$

where I is the moment of inertia of the pendulum.

In order to deal in detail with the function F which represents the energy dissipated per cycle, I propose to introduce as a hypothesis a law of internal friction with which the experiments turn out to be in agreement. Let us

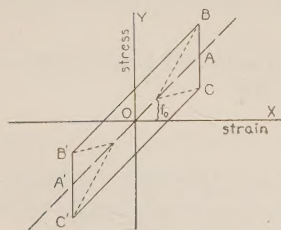


Fig. 3. Hypothetical stress-cycle.

assume first that the type of stress is either simple shear or simple tension, so that the strain-energy can be discussed in terms of one variable stress. In Fig. 3 let OX be the axis of strains and OY the axis of stresses, and let OA be the curve of stress vs. strain for this material, so that in the absence of internal friction a cycle of stress would be OAOA'O. Assume that the actual history of a stress cycle is the parallelogram B'BCC'B, during which the work done per unit volume is equal to

the area of the parallelogram. Further let the rule for determining the half-height AB be as follows

$$AB = \phi(f - f_0) \quad (6)$$

where ϕ is the coefficient of internal friction, defined by this equation, and f_0 is a limiting stress, below which the friction stress AB is negligible or follows a different law. It is now plain that the energy per unit volume dissipated during one cycle will be $4\phi(f^2 - f_0f)/E$, and for the entire specimen

$$F = \iiint 4\phi(f^2 - f_0f)dv/E \quad (7)$$

In the case of a hollow tubular specimen under torsion all parts of the metal are subjected to nearly the same stresses, so that the volume integrals of Eqs; (4) and (7) may be expressed as simple products, and we have

$$F = 8\phi W(1 - f_0/f) \text{ or } F = 8\phi W(1 - A_0/A) \quad (8)$$

where A_0 is the amplitude corresponding to the stress f_0 . Substituting in Eq. (8) the values of F given by Eq. (3), and of W given by Eq. (5), we finally obtain

$$L = 4\pi^2 I n^2 \phi (A - A_0) \quad (9)$$

Thus, according to our assumption, a curve where L is plotted against A should be a straight line intersecting the A -axis at a point A_0 .

The situation is more complicated if, as is the case in bending, all parts of the specimen are not subjected to the same stress-cycles. Here the curve L vs. A will not be a straight line; but it will intersect the A -axis at A_0 , and its slope will approximate the value given by Eq. (9) for large amplitudes.

If we still further limit ourselves to cases where $f_0 = 0$, which is often true for specimens without fatigue history, our cancellations are performed under the sign of integration and Eq. (9) becomes

$$L = 4\pi^2 I n^2 A \phi \quad (10)$$

which will then be true either for torsion or bending.

It only remains to note that the special restriction $f_0 = 0$ renders our present law of internal friction identical with that used by Kimball and Lovell and others; i.e., the energy dissipated per cycle is proportional to the stress-amplitude squared. In fact, the energy dissipated per cycle is put by Kimball equal to ξf^2 where ξ is Kimball's constant of internal friction, from which it is evident that

$$\xi = 4\phi/E \quad (11)$$

The points in favor of ϕ as a coefficient of internal friction are (1) that it is dimensionless, and hence independent of systems of units; (2) that its definition is closely analogous to that of the ordinary friction coefficient, and (3) that its use in conjunction with Eqs. (7) and (9) does appear to represent the facts as developed in the ensuing experimental results.

The form of the tuning curve for a vibration performed under the above conditions may be obtained by calculating the work done during two suc-

cessive quarter swings, and eliminating $\sin \alpha$ from the resulting equations. It turns out to be

$$a = L / 2\pi^2 I [n_r^2(1 + 2\phi) - n^2] \quad (12)$$

where a is the amplitude corresponding to frequency n , n_r is resonance frequency, and the brackets indicate that the positive value of the quantity within them is to be used.

There is a simple relation between ϕ and the logarithmic decrement δ . If we define δ by the equation $A = A_0 e^{-n\delta}$, where A is the amplitude of free vibration after nt complete vibrations which started at amplitude A_0 , it turns out that if $f_0 = 0$,

$$\delta = 4\phi \quad (13)$$

EXPERIMENTS AND RESULTS

Hollow specimens of the following metals were used: Armco iron (a nearly pure metal); Navy brass, cold drawn; phosphor bronze, annealed. In addition there were solid specimens of mild steel, copper, aluminum, Monel metal, Tobin bronze, and machine steel. The hollow specimens, except the Armco iron, had a fatigue history prior to these experiments; the solid specimens, except the mild steel, were new. The procedure was to measure the specimen, insert it in the apparatus, and then, starting with the smallest possible current in the windings, adjust the frequency of the exciting impulse until resonance was obtained. The amplitude of the resonance peak A was noted,

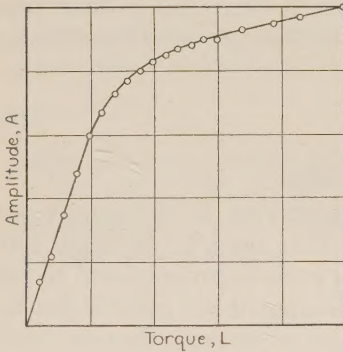


Fig. 4. Solid specimen of mild steel (torsion).

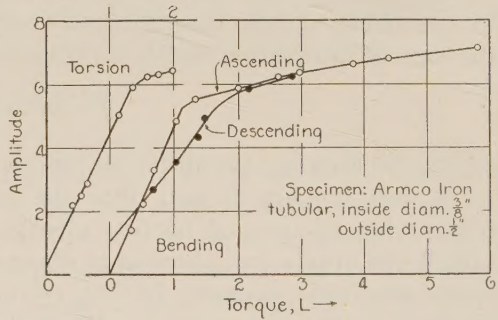


Fig. 5. Torsion and bending curves for armco iron.

along with the frequency n , and the exciting couple L . Then L was increased and the process was repeated. As soon as it was plain that the upper limit of simple friction had been passed, the pendulum, magnets and mirror were adjusted for the other mode of vibration, and the process repeated.

An analysis of numerous curves representing the relation between torque and amplitude showed in all cases the features demanded by the theoretical deductions given above. A certain range of amplitudes (sometimes rather

short) was always found where the simple law of friction held true, as evidenced by the curve's being linear. Nearly always, at least in the initial observations, the threshold stress f_0 was zero indicating that the dissipated energy was proportional to the square of the stress-amplitude. Fig. 4 illus-

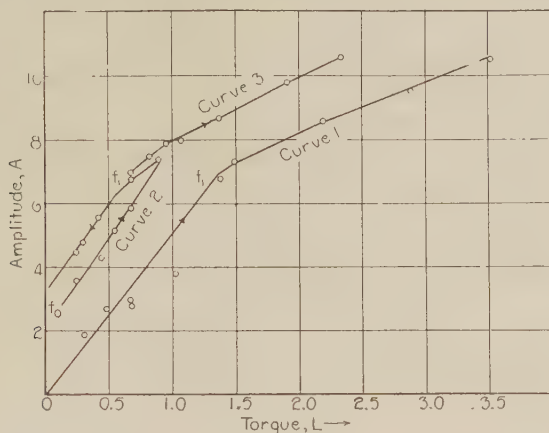


Fig. 6. Effect of repeated vibration on brass (torsion).

trates such a result. In this case the specimen was solid and the upper limit f_1 of the simple friction range is not as clearly defined as in Fig. 5, which represents results both for torsion and bending of a hollow specimen.

The reversals of high stress-range for the right-hand curve of Fig. 5 apparently resulted in a change of properties as indicated by the "descending"

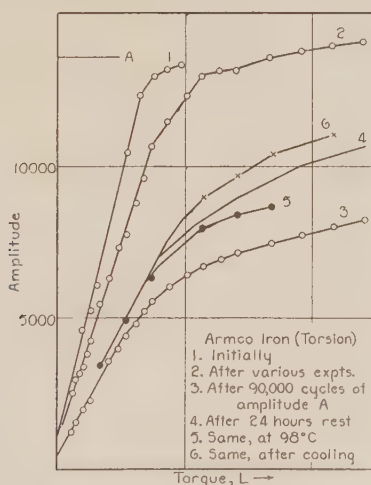


Fig. 7. Armco Iron (Torsion): 1. Initially; 2. after various expts.; 3. after 90/000 cycles of amplitude "A"; 4. after 24 hours rest; 5. same, at 98°C; 6. same, after cooling.

portion of this curve. Here there has been an increase of ϕ , indicated by the flatter slope; and the threshold stress f_0 has made its appearance. The limit f_1 is unchanged.

Fig. 6 shows the results of torsion experiments on a hollow brass specimen. Here the increase of f_0 with fatigue is very plain, but ϕ remains nearly constant, as does f_1 .

Fig. 7 shows the results of a whole series of torsion experiments on Armco iron, made some days after the preceding ones. It will be seen that the fatigue treatment after curve 2 was taken has decreased f_1 to a marked degree, has increased ϕ , and has decreased f_0 .

The only experiment involving the effect of temperature is shown by curve 5 of Fig. 7. A temperature of 98° has not changed ϕ or f_1 , but has decidedly increased the dissipation above f_1 . The same temperature has a very marked effect on the elastic constants, and a like change in ϕ would easily have been observed.

The threshold stress f_0 seems on the whole to be a feature of overstrained or fatigued metals. It was observed initially only in three specimens, steel, Tobin bronze, and Monel. The first two of these were known to have been cold-rolled. That it may disappear in some cases after a period of rest is indicated by the curves for specimens used in 1925, e.g., Fig. 4.

The upper limit f_1 of the straight line relation is in some cases about equal to the elastic limit, in others far below it. Values of this constant are tabulated in the last column of Table 1. It is recorded as a shear stress, and it is noteworthy that it has practically the same value whether derived from the twisting or the bending curve.

TABLE I

No.	Material	Friction coefficients		$\frac{\phi_B}{\phi_S}$	$\frac{E}{3N}$	Limiting shear (Kg. per cm ²)	
		ϕ_S (Torsion)	ϕ_B (Bending)			Torsion	Bending
1	Aluminum	0.0075	0.0065	0.86	0.81	169	162
2	Navy Brass*	.0060	.0048	.79	.82	623	667
3	Copper††	.0076	.0063	.83	.82	76	41
4	Armco Iron	.0065	.0051	.78	.86	810	890
4a	Armco Iron*	.0084	.0087	1.04	.81	—	—
5	Monel	.0063	.0053	.84	.82	114	137
6	Cold-rolled Steel†	.0030	.0051	1.70	.75	—	—
6a	Cold-rolled steel*	.0059	.0062	1.05	.75	693	745
7	Tobin Bronze†	.0060	.0068	1.13	.82	142	109
8	Phosphor Bronze†	.0018	—	—	—	570	—
8a	Phosphor Bronze*	.0083	.0027	.32	.82	194	218
9	Mild Steel*	.0140	—	—	—	1180	—

* Fatigue history in laboratory

† Cold-worked material

†† Annealed material

RELATION OF INTERNAL FRICTION TO SHEARING STRAIN

One object of the study was to determine what relation could be found between the coefficients of friction for torsion and bending. The experimental results are given in Table I. It will be noted that for the first five specimens the ratio ϕ_B/ϕ_S is approximately equal to the fraction $E/3N$, where E is Young's modulus and N is the rigidity modulus, the ratio being determined

experimentally. The energy of any state of strain may be represented as the sum of two parts, i.e., shear energy and dilatation energy. In torsion the energy is 100 percent shear; in bending the fraction of the total energy due to shear is $E/3N$. Thus, it is plain that if ϕ_B/ϕ_S is equal to $E/3N$, it is only the shear energy which is dissipated by internal friction.

The remaining five observations do not, at a first glance, confirm the conclusion that the friction is associated with shear only, but reasons which readily account for this discrepancy are close at hand. It will be seen from a later paragraph that continued vibration at high amplitudes (i.e., fatigue) increases ϕ . There is evidence to show that bending fatigue increases bending friction without affecting twisting friction, and vice versa. Recalling that the planes of maximum shear for bending are at an angle of 45° with those for twisting, we perceive that all these facts arrange themselves coherently if we view the friction as localized in certain planes of certain crystal grains, and as being altered by repetitions of the gliding along those planes. As an example, the phosphor-bronze specimen had received a great amount of twisting fatigue prior to these experiments, but no bending fatigue. Entry 8 in Table 1 gives the value of ϕ_S for this metal at the commencement of work in 1927. It had already a fatigue history dating back to 1925. Before the observations under entry 8a were made, it received an added, severe amount of twisting fatigue, resulting in the increase of ϕ_S from 0.0018 to 0.0083; but at this latter time ϕ_B was only 0.0027. Other observations, which space does not permit including, bear out the same conclusions.

TABLE II. Values of ϕ calculated by Eq. (11) from values presented by Kimball and Lovell.

Material	Coefficients of Internal Friction		Experimenter
	ϕ_S	ϕ_B	
Aluminum		0.0008	K
Brass		.0012	K
Swedish iron		.0020	K
Phosphor-bronze		.0008	K
Monel metal		.0004	K
Cold-rolled steel		.0012	K
Steel	0.00060	.00048	V
Brass	.00017	.00008	V
Aluminum	.00021	.00150	V

K—Kimball & Lovell; V—Voigt

Table II presents a series of values of ϕ calculated by Eq. (11) from values presented by Kimball and Lovell¹ representing their own results as well as those of other experimenters. The values are on the whole lower than those obtained by the writer. The discrepancy can be accounted for if we assume that the specimens used by the other workers had a fairly large threshold stress f_0 , since the calculations of Table II presuppose that this is zero. It is also certain that some of the specimens of the present paper, due to prior fatigue history, possessed unusually large coefficients. It is also true that some experimenters have worked at *very* low stress ranges, precisely those which are beyond the reach of study by the present method.

SUMMARY

1. It is found that for a certain range of stresses the energy dissipated per unit volume per stress-cycle is proportional to $f(f-f_0)$ where f is the stress-amplitude and f_0 is a threshold stress below which the friction is zero or of a very small order.

2. The threshold f_0 is a function of the mechanical history, being probably zero in unstrained material, increased by cold-working or fatigue, decreased again by severe fatigue.

3. The upper limit f_1 of the friction range is affected to a much less extent by physical history, but severe fatigue makes it lower. Above f_1 the energy dissipated increases faster than below it, and is no longer a quadratic function of the amplitude.

4. The dissipation of energy by internal friction in metals without history of overstrain or fatigue seems only to accompany shearing stress. Pure dilatation is frictionless.

5. Severe fatigue increases the coefficient of internal friction. This change when due to torsion proceeds to some extent independently of the same effect caused by bending (which involves a shear stress on planes of different orientation than does torsion), and vice versa.

In concluding I wish to thank Professor A. G. Christie for the services of his shop and mechanic, and both him and Professor J. S. Ames for kindly interest in the work.

THE JOHNS HOPKINS UNIVERSITY,
April 18, 1928.

VITA.

Robert Hawthorne Canfield, the son of John Grier and Ada (Drabelle) Canfield, was born in Denver, Colorado, on June 14, 1894. He received his early education in the public schools of that city. Later he attended the University of Colorado and received the degree B. S. (C. E.) in 1915. After four years spent in business and in the army he returned to the University of Colorado where he taught and studied until 1923, receiving in that year the degrees M. S. and C. E. He then entered the Johns Hopkins University as a graduate student in Physics, taking work under Professors Ames, Cohen, Morley, Murnaghan, Pfund and Wood.

